

1. En lais $y = y(t, r)$ $\vec{j}_0 = j_0(r) \vec{u}_r - \oint \vec{j}_0 \cdot d\vec{S} + \iiint \rho \, d\tau = \iiint \frac{\partial \rho}{\partial t} \, d\tau$ (*) (permanent).

• pour $r < R$ (*) $\Rightarrow -j_{0r} 4\pi r^2 + \lambda \cdot \frac{4}{3} \pi r^3 = 0 \Rightarrow j_{0r} = \frac{\lambda r}{3}$ pour $r < R$

• pour $r > R$ (*) $\Rightarrow -j_{0r} 4\pi r^2 + \lambda \frac{4\pi R^3}{3} = 0 \Rightarrow j_{0r} = \frac{\lambda R^3}{3r^2}$ pour $r > R$

2. Pour $r > R$ $\vec{j}_{0r} = \frac{\lambda R^3}{3r^2} \vec{u}_r = -\kappa_e \frac{dT}{dr} \vec{u}_r \Rightarrow \int_0^r \frac{dT}{dr} dr = -\frac{\lambda R^3}{3\kappa_e} \int_0^r \frac{dr}{r^3}$

$\Rightarrow T(r) - T_0 = \frac{\lambda R^3}{3\kappa_e} \left[\frac{1}{r} - \frac{1}{R} \right] \Rightarrow T(r) = T_0 + \frac{\lambda R^3}{3\kappa_e r}$ si $r > R$

donc $T_e(R) = T_0 + \frac{\lambda R^2}{3\kappa_e}$

Conservation de l'énergie $j_{0r} = -h(T_e(R) - T(R)) = \frac{\lambda R}{3}$

d'où $T(R) = \frac{\lambda R}{3h} + T_e(R) = T_0 + \frac{\lambda R^2}{3\kappa_e} + \frac{\lambda R}{3h} = T(R)$

puis $j_{0r} = -\kappa \frac{dT}{dr} \Rightarrow \frac{dT}{dr} = -\frac{\lambda r}{3\kappa} \quad (r < R) \Rightarrow \int_R^r \frac{dT}{dr} dr = -\int_R^r \frac{\lambda r}{3\kappa} dr$

$\Rightarrow T(r) - T(R) = -\frac{\lambda}{6\kappa} (r^2 - R^2) \Rightarrow$

$T(r) = T(R) + \frac{\lambda R^2}{6\kappa} - \frac{\lambda r^2}{6\kappa} \quad r < R$

$\Rightarrow T(r) = \left[T_0 + \frac{\lambda R^2}{3\kappa_e} + \frac{\lambda R}{3h} + \frac{\lambda R^2}{6\kappa} \right] - \frac{\lambda r^2}{6\kappa}, \quad r < R$

