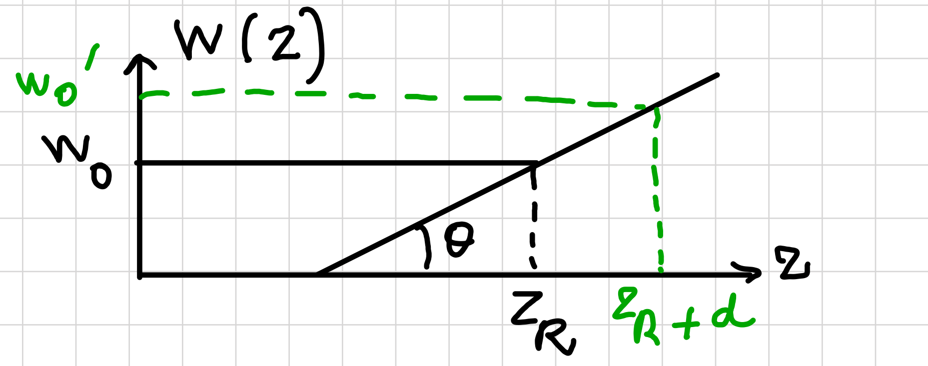
TD Laser

Ex 1 ① $\lambda = 632{,}8\ \text{nm}$, $w_0 = 0{,}60\ \text{mm}$

ⓐ



$d = 0{,}01\ \text{mm}$

× $\boxed{\theta = \dfrac{\lambda}{\pi w_0}}$

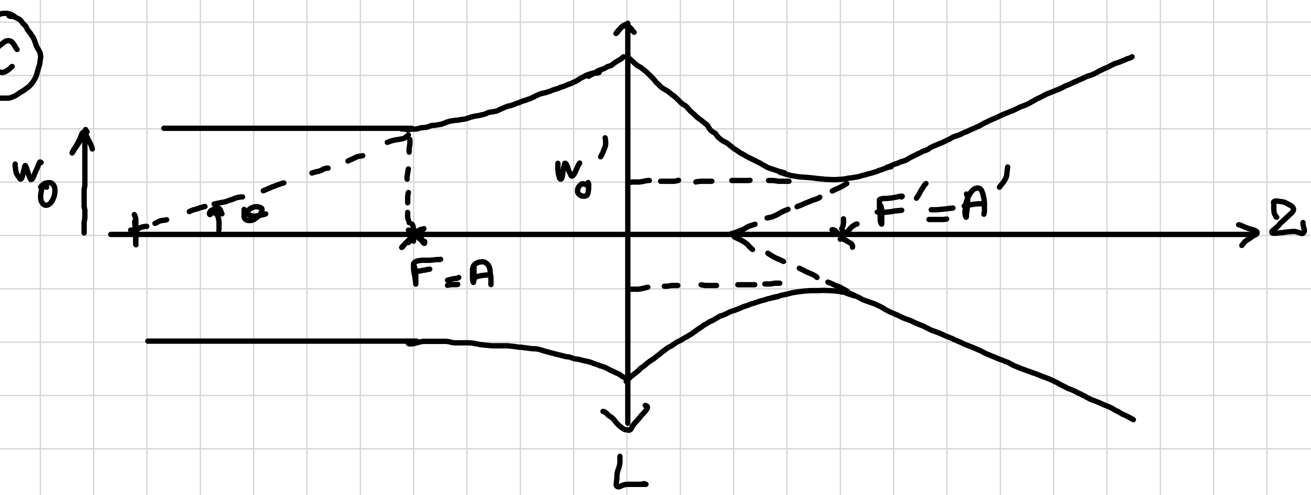
AN : $\theta = 0{,}34\ \text{mrad}$

× $\boxed{z_R = \dfrac{w_0}{\theta}} = 1{,}79\ \text{m}$

ⓑ $\theta = \dfrac{w_0}{z_R} = \dfrac{w_0'}{z_R + d}$

$$\Rightarrow w_0' = w_0 \left(\frac{z_R + d}{z_R}\right) = w_0 \left(1 + \underbrace{\frac{d}{z_R}}_{\ll 1}\right) \simeq w_0$$

ⓒ



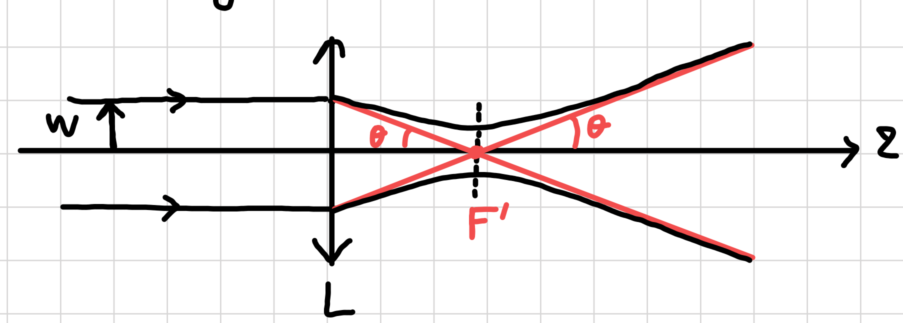
× $\sigma = \overline{FA} = 0 \Rightarrow \sigma' = \overline{F'A'} = 0$

× $w_0' = \dfrac{w_0 f'}{\sqrt{\sigma^2 + z_R^2}} = \dfrac{w_0 f'}{z_R}$ avec $\begin{cases} w_0 = 0{,}60\ \text{mm} \text{ et } f' = 10\ \text{mm} \\ z_R = 1{,}79\ \text{m} \end{cases}$

AN : $w_0' = 33{,}6\ \mu\text{m}$

× $\theta' = \dfrac{\lambda}{\pi w_0'} = 6{,}00\ \text{mrad}$

②

× 

× faisceau $\simeq$ cylindrique en entrée

$\Rightarrow \sigma = \overline{FA} \gg z_R$
$\gg f'$

$$\Rightarrow \sigma' = \frac{f'\sigma}{\sigma^2 + z_R^2} = 0 \Rightarrow \boxed{F' = A'}$$

$\times$ $\theta = \dfrac{w}{f'}$ et $\begin{cases} \theta = \dfrac{\lambda}{\pi w_0'}, \\ \theta = \dfrac{w_0'}{z_R'} \end{cases}$ $\Rightarrow \theta^2 = \dfrac{\lambda}{\pi z_R'}, = \dfrac{w^2}{f'^2}$

$$\Rightarrow \boxed{f'^2 = \frac{\pi w^2 z_R'}{\lambda}}$$

AN : $\underline{f' = 14,9 \text{ cm}}$

3 a) $f = n\left(\dfrac{c}{2d}\right) \Rightarrow \boxed{\Delta f = \dfrac{c}{2d}}$ AN : $\Delta f = 7,5 \times 10^8$ Hz $= 750$ MHz

b) $\Delta f = \dfrac{c}{2d} > \Delta f_{\text{filtre}}$

Avec $\boxed{\Delta f_{\text{filtre}} = \dfrac{c}{\lambda_0^2} \Delta\lambda_{\text{filtre}}}$

$$\Rightarrow \boxed{d < \frac{\lambda_0^2}{2\Delta\lambda_{\text{filtre}}}}$$ $\underline{\text{AN}}$: $d < 200\,\mu\text{m}$.

4 a) • Intensité faisceau Gaussien :

$$I(z,r) = A_0 \frac{w_0^2}{w(z)^2} \exp\left(-\frac{2r^2}{w(z)^2}\right)$$

$$\Rightarrow P_{tot} = \int_0^{2\pi}\int_0^{\infty} I \, r\,dr\,d\theta = A_0 \frac{w_0^2}{w(z)^2} \times 2\pi \int_0^{\infty} r e^{-\frac{2r^2}{w^2}} dr$$

$$= 2\pi A_0 \frac{w_0^2}{w^2}\left[-w^2 e^{-2r^2/w^2}\right]_0^{\infty}$$

$$\boxed{P_{tot} = 2\pi A_0 w_0^2}$$

• $P_e = \displaystyle\int_0^{2\pi}\int_0^{e} I \, r\,dr\,d\theta = \frac{A_0 w_0^2}{w(z)^2} \times 2\pi \int_0^{e} r e^{-\frac{2r^2}{w^2}} dr$

$$\mathcal{P}_e = 2\pi A_0 \frac{w_0^2}{w^2}\left[-w^2 e^{-2r^2/w^2}\right]_0^e$$

$$\boxed{\mathcal{P}_e = 2\pi A_0 w^2 \left[1 - e^{-2e^2/w^2}\right]}$$

D'où : $\boxed{\dfrac{\mathcal{P}_e}{\mathcal{P}_{tot}} = 1 - e^{-2e^2/w^2}}$

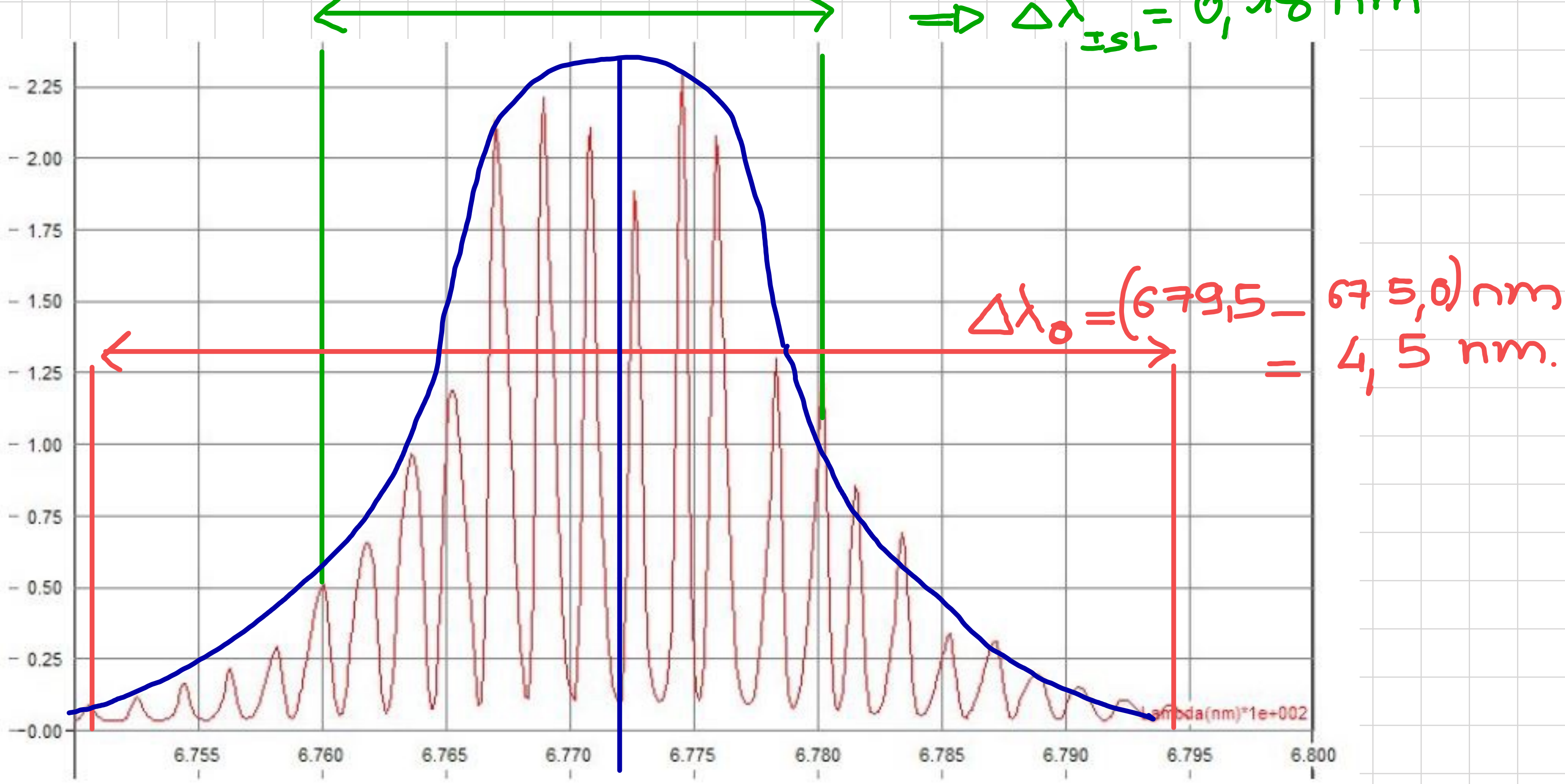
b) AN :

$$\begin{cases} e = 0{,}5w \Rightarrow \dfrac{\mathcal{P}_e}{\mathcal{P}_{tot}} = 1 - e^{-1/2} = \\ e = 0{,}75w \Rightarrow \dfrac{\mathcal{P}_e}{\mathcal{P}_{tot}} = 1 - e^{-2\times\frac{9}{16}} \\ e = w \Rightarrow \dfrac{\mathcal{P}_e}{\mathcal{P}_{tot}} = 1 - e^{-2} \\ e = 2w \Rightarrow \dfrac{\mathcal{P}_e}{\mathcal{P}_{tot}} = 1 - e^{-8} \end{cases}$$

Exercice 2

$11 \times \Delta\lambda_{ISL} = (678{,}0 - 676{,}0)\,nm = 2\,nm$

$\Rightarrow \Delta\lambda_{ISL} = 0{,}18\,nm$



$\lambda_0 = 677{,}2\,nm$

* où $\begin{cases} \Delta\lambda_{ISL} = \text{intervalle spectrale libre (modes longitudinaux)} \\ \Delta\nu_{ISL} = \dfrac{c}{2nd} = \dfrac{c}{\lambda_0^2}\,\Delta\lambda_{ISL} \Rightarrow \boxed{\Delta\lambda_{ISL} = \dfrac{\lambda_0^2}{2nd}} \end{cases}$

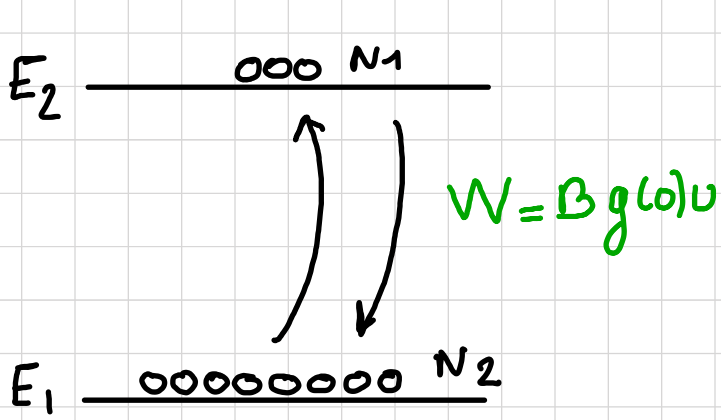
$\Rightarrow \boxed{d = \dfrac{\lambda_0^2}{2n\,\Delta\lambda_{ISL}}}$ AN $d = 3{,}5\,mm$

et $\Delta\nu_{ISL} = \dfrac{c}{\lambda_0^2}\,\Delta\lambda_{ISL} = \underline{118\,GHz}$

x $\boxed{\Delta\nu_0 = \dfrac{c}{\lambda_0^2}\,\Delta\lambda_0} = \underline{2{,}94\,THz}$

x $\boxed{\Delta\nu_0 \times \tau_c = 1} \Rightarrow \tau_c = 3{,}40\,ps$ et dc $l_c = c\tau_c = \underline{102\,\mu m}$

Exercice 3



② $\begin{cases} \dfrac{dN_1}{dt} = AN_2 - BN_1 u g(0) + BN_2 u g(0) \\ \dfrac{dN_2}{dt} = -AN_2 + BN_1 u g(0) + BN_2 u g(0) \end{cases}$

RS : $\begin{cases} AN_{2eq} - BN_{1eq} u_{eq}\, g(0) + BN_{2eq} u_{eq}\, g(0) \\ N = N_{1eq} + N_{2eq} \end{cases}$

$$\Rightarrow \begin{cases} N_{1eq} = N \left(\dfrac{A + B u_{eq}\, g(0)}{A + 2B_{eq}\, u_{eq}\, g(0)} \right) \\ N_{2eq} = N \left(\dfrac{B u_{eq}\, g(0)}{A + 2B_{eq}\, u_{eq}\, g(0)} \right) \end{cases}$$

on ne peut pas avoir $N_2 > N_1$ (au mieux $N_2 = N_1 = \frac{N}{2}$)

③ x Facteur de Boltzmann :

$$\frac{N_{1eq}}{N_{2eq}} = \exp\left(\frac{E_2 - E_1}{k_B T}\right) > 1$$

x $Bug(0) \gg A \Rightarrow$ Le gaz est placé ds une densité em intense $\Rightarrow$ bcp d'absorption et d'émission stimulée (pas d'émission spontanée)

④ x Transition optique $\Rightarrow E_2 - E_1 \gg k_B T$

$$\Rightarrow \frac{N_{1eq}}{N_{2eq}} = \exp\left(\frac{E_2 - E_1}{k_B T}\right) \gg 1$$

AN $\begin{cases} T = 300K \\ \lambda = 60nm \end{cases}$

$N_2 / N_1 = 2.10^{-35} \ll 1$

x OR : $\dfrac{N_{1eq}}{N_{2eq}} = \dfrac{A + B u_{eq}\, g(0)}{B u_{eq}\, g(0)} \gg 1 \Rightarrow \dfrac{A}{B u_{eq}\, g(0)} \gg 1$

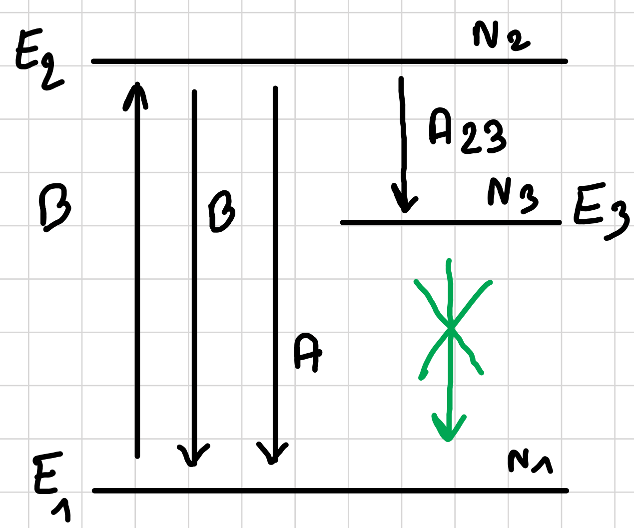
$\Rightarrow$ Cavité "diluée" : c'est l'émission spontanée qui prédomine

(5) Suite de la question 4 : $A \gg B u g(0)$

$$\begin{cases} \dfrac{dN_1}{dt} = A N_2 = A(N - N_1) \\ \dfrac{dN_2}{dt} = -A N_2 \end{cases} \qquad \begin{cases} \dfrac{dN_1}{dt} + A N_1 = A N \\ \dfrac{dN_2}{dt} + A N_2 = 0 \end{cases}$$

$$\Rightarrow \boxed{\tau_2 = \frac{1}{A} = \text{durée de vie radiative}}$$

(6)



(b) on ne tient compte que de l'émission spontanée :

$$\begin{cases} \dfrac{dN_1}{dt} = A N_2 \\ \dfrac{dN_2}{dt} = -A N_2 - A_{23} N_2 \\ \qquad = -(A + A_{23}) N_2 \end{cases}$$

$$\begin{cases} \dfrac{dN_1}{dt} = A N_2 \\ \dfrac{dN_2}{dt} + \underbrace{(A + A_{23})}_{1/\tau_2} N_2 = 0 \end{cases}$$

(c)
$$\begin{cases} \dfrac{dN_1}{dt} = B u g(0) (N_2 - N_1) + A N_2 & (1) \\ \dfrac{dN_2}{dt} = -B u g(0) (N_2 - N_1) - (A + A_{23}) N_2 & (2) \\ \dfrac{dN_3}{dt} = -A_{23} N_2 & (3) \end{cases}$$

Equilibre : $\dfrac{dN_3}{dt} = 0 \overset{(3)}{\Rightarrow} N_2 = 0 \overset{(1)}{\Rightarrow} N_1 = 0$ et dc $\boxed{N_3 = N}$

Inversion

Interprétation : on remplit le niveau 3 car les probabilité de transition st $\neq 0$ sauf $3 \to 1$