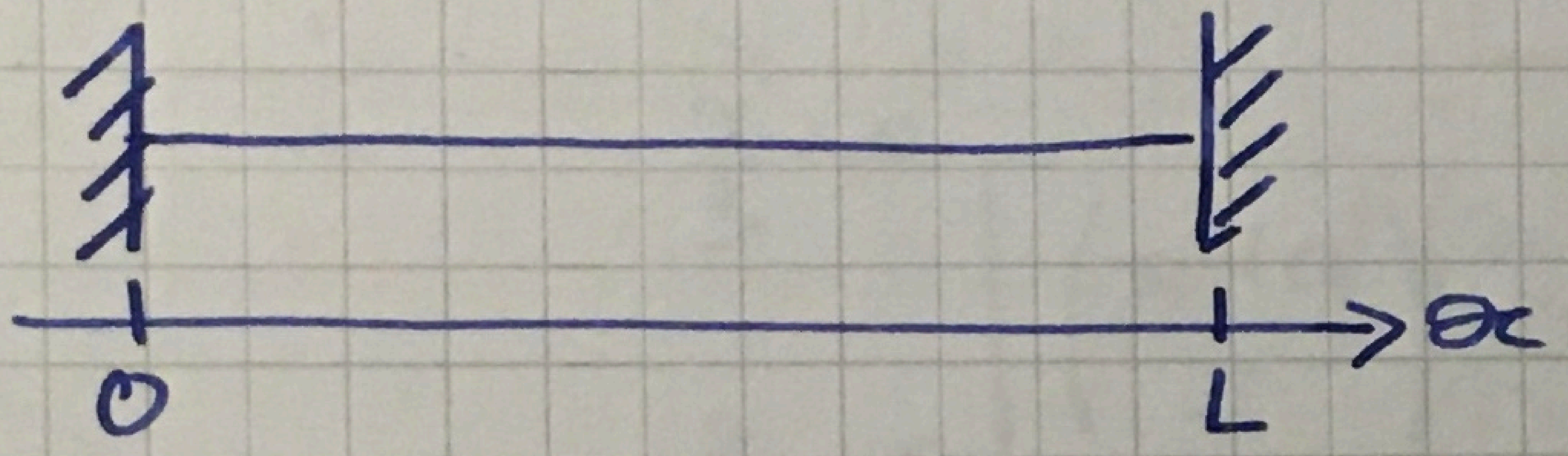
Ex 13



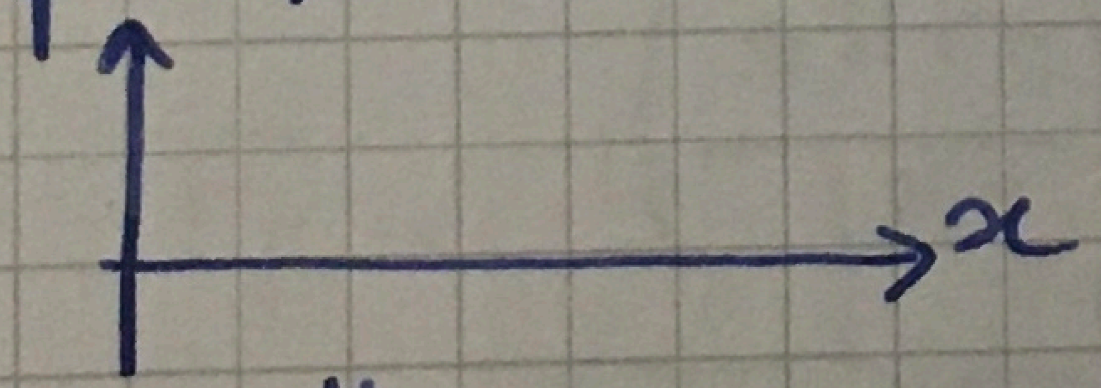
$$y(x,t) = \sum_n \left(A_n \cos \frac{n\pi c}{L} t + B_n \sin \frac{n\pi c t}{L}\right) \sin\left(\frac{n\pi x}{L}\right)$$

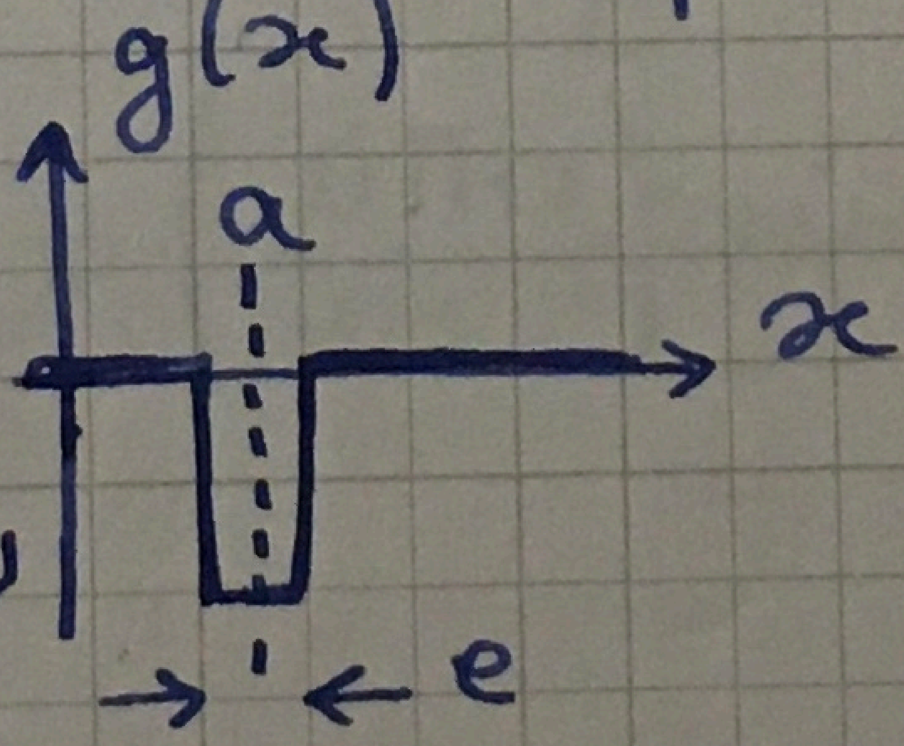
$$y(x,0) = \sum_n A_n \sin\left(n\pi \frac{x}{L}\right) = f(x)$$

$$\left.\frac{\partial y}{\partial x}\right)_{(x,0)} = \sum_n B_n \cdot \frac{n\pi c}{L} \cdot \sin\left(n\pi \frac{x}{L}\right) = g(x)$$

<u>Corde de piano</u> $\Rightarrow$ corde frappée

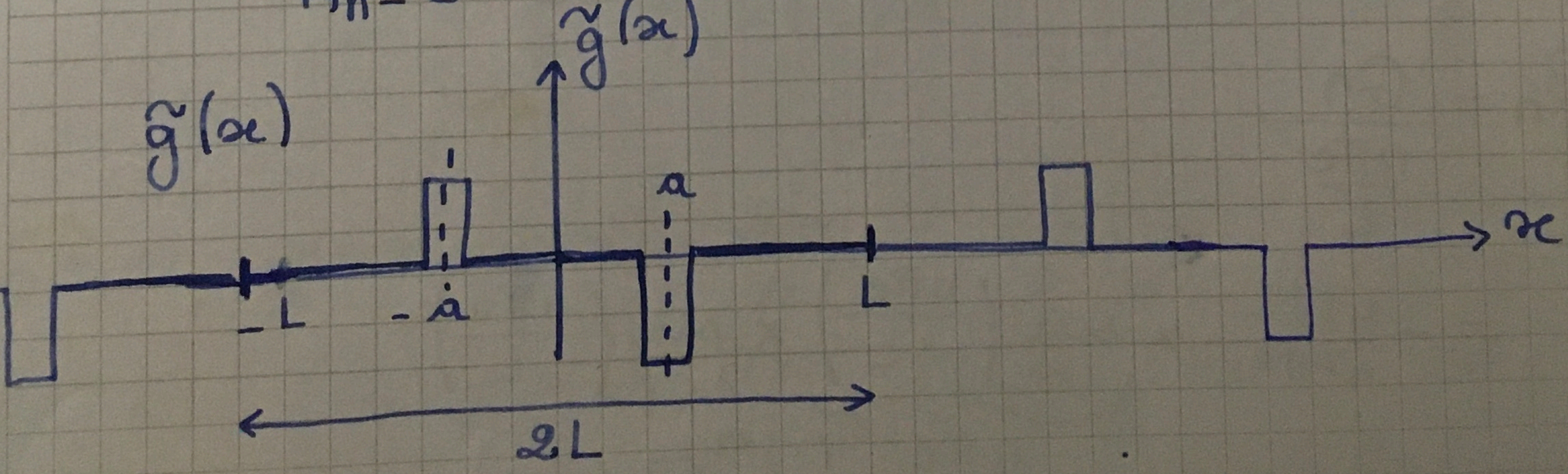
$f(x) = 0$





$\Downarrow$

$A_n = 0$



$\tilde{g}(x)$ est $2L$ périodique impaire

$$\tilde{g}(x) = \sum_n G_n \sin\left(\frac{2\pi n x}{2L}\right) \Rightarrow \boxed{G_n = \frac{n\pi c}{L} B_n}$$

$$G_n = \frac{4}{2L} \int_0^{\frac{2L}{2}} \tilde{g}(x) \sin \frac{n\pi x}{L} dx$$

$$= \frac{2}{L} \int_{a-\frac{e}{2}}^{a+\frac{e}{2}} -u \sin\left(\frac{n\pi x}{L}\right) dx$$

$$= -\frac{2U}{L} \left[\frac{-L}{n\pi} \cos\left(\frac{n\pi x}{L}\right)\right]_{a-\frac{e}{2}}^{a+\frac{e}{2}}$$

$$= \frac{2U}{n\pi} \left[\cos\left(\frac{n\pi}{L}\left(a+\frac{e}{2}\right)\right) - \cos\left(\frac{n\pi}{L}\left(a-\frac{e}{2}\right)\right)\right]$$

$$= \frac{2U}{n\pi} (-2) \sin\left(\frac{n\pi}{L}\left(\frac{a+\cancel{\frac{e}{2}}+a-\cancel{\frac{e}{2}}}{2}\right)\right)$$

$$\times \sin\left(\frac{n\pi}{L}\left(\frac{a+\frac{e}{2}-a+\frac{e}{2}}{2}\right)\right)$$

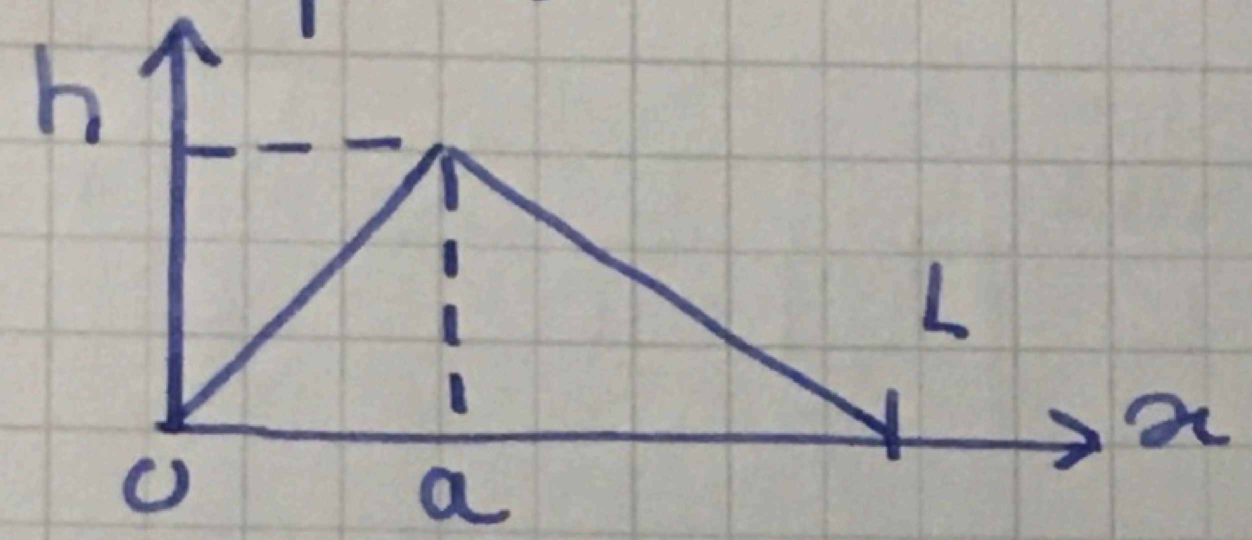
$$= -\frac{4U}{n\pi} \times \sin\left(\frac{n\pi a}{L}\right) \times \underbrace{\sin\left(\frac{n\pi e}{2L}\right)}_{\simeq \frac{n\pi e}{2L}} \text{ (1}^{er}\text{ termes)}$$

$$B_n = \frac{G_n}{\frac{n\pi c}{L}}$$

$$= -\frac{4UL}{n^2\pi^2 c} \times \underbrace{\sin\left(\frac{n\pi e}{2L}\right)}_{\frac{n\pi e}{2L}} \times \sin\left(\frac{n\pi a}{L}\right) \text{ (1}^{er}\text{ termes)}$$

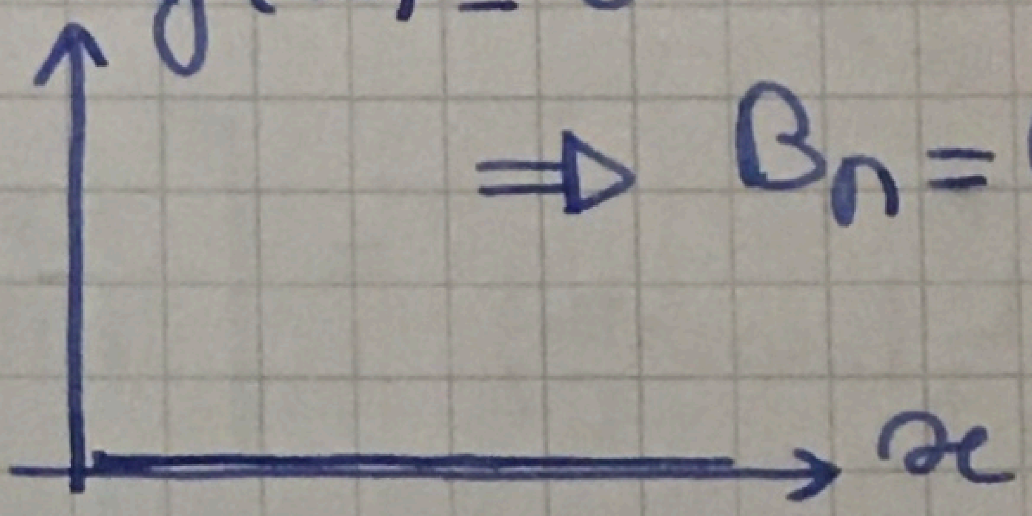
$$\boxed{B_n \simeq -\frac{2Ue}{n\pi c} \sin\left(\frac{n\pi a}{L}\right)} \quad \propto \frac{1}{n}$$

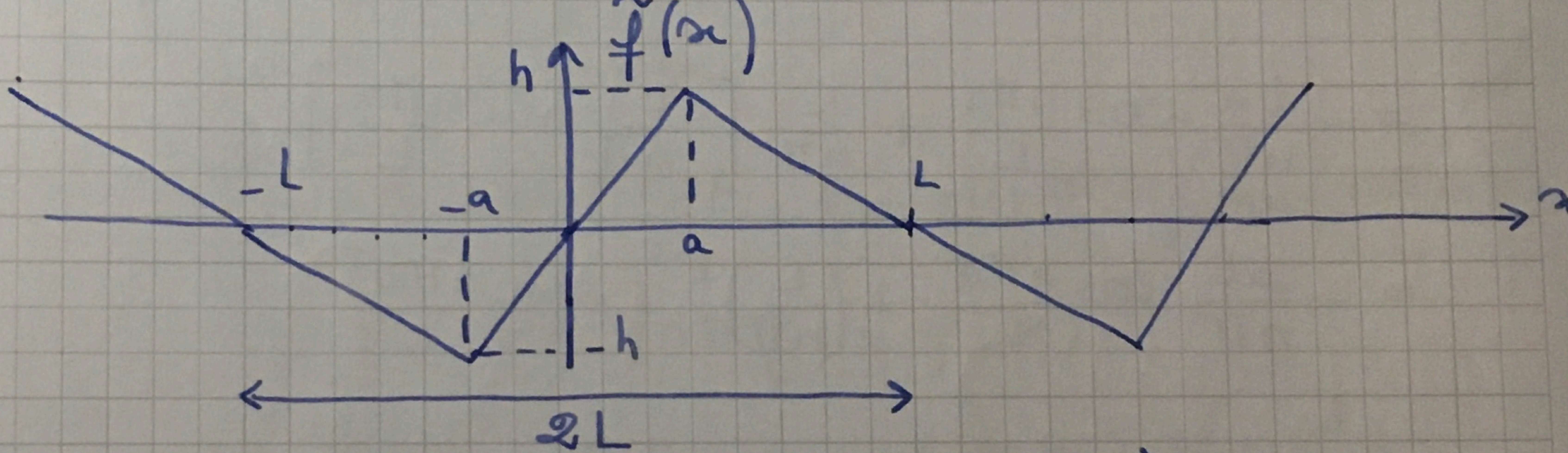
Corde de clavecin $\Rightarrow$ pincée



$g(x) = 0$

$\Rightarrow B_n = 0$





$\tilde{f}(x)$ est $2L$ périodique impaire

$$\Rightarrow \tilde{f}(x) = \sum_n F_n \sin \frac{n\pi x}{L} \Rightarrow \boxed{A_n = F_n}$$

$$F_n = \frac{4}{2L} \int_0^{\frac{2L}{2}} \tilde{f}(x) \sin \frac{n\pi x}{L}\, dx$$

$$= \frac{2}{L} \int_0^{L} \tilde{f}(x) \sin \frac{n\pi x}{L}\, dx$$

$0 < x < a$: $\tilde{f}(x) = f(x) = \frac{hx}{a}$.

$a < x < L$ $\tilde{f}(x) = f(x) = \frac{h}{(L-a)}(L-x)$

$$F_n = \frac{2}{L}\left[\int_0^a \frac{hx}{a} \sin \frac{n\pi x}{L}\, dx + \int_a^L \frac{h}{L-a}(L-x) \sin \frac{n\pi x}{L}\, dx\right]$$

$$\bullet \int_0^a \underbrace{x}_{u} \underbrace{\sin\frac{n\pi x}{L}}_{v'} dx = \Big[uv \Big]_a^a - \int_0^a u' v \, dx$$

$$u' = 1 \qquad v = \frac{L}{n\pi}\left(-\cos\frac{n\pi x}{L}\right)$$

$$= \left[-\frac{Lx}{n\pi} \cos\frac{n\pi x}{L} \right]_0^a + \int_0^a \frac{L}{n\pi} \cos\frac{n\pi x}{L} dx$$

$$= -\frac{La}{n\pi} \cos\frac{n\pi a}{L} + \frac{L}{n\pi}\left[\frac{L}{n\pi} \sin\frac{n\pi x}{L} \right]_0^a$$

$$= -\frac{aL}{n\pi} \cos\frac{n\pi a}{L} + \left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi a}{L}$$

$$\bullet \int_a^L \underbrace{(L - x)}_{u} \underbrace{\sin\frac{n\pi x}{L}}_{v} dx$$

$$u' = -1$$

$$v = \frac{L}{n\pi}\left(-\cos\frac{n\pi x}{L}\right)$$

$$= \left[-\frac{(L-x)L}{n\pi} \cos\frac{n\pi x}{L} \right]_a^L - \int_a^L \frac{L}{n\pi} \cos\frac{n\pi x}{L} dx$$

$$= -\frac{L}{n\pi}\left[0 - (L-a) \cos\frac{n\pi a}{L} \right] - \left(\frac{L}{n\pi}\right)^2 \left[\underbrace{\sin n\pi}_{0} - \sin\frac{n\pi a}{L} \right]$$

$$= \frac{L(L-a)}{n\pi} \cos\frac{n\pi a}{L} + \left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi a}{L}$$

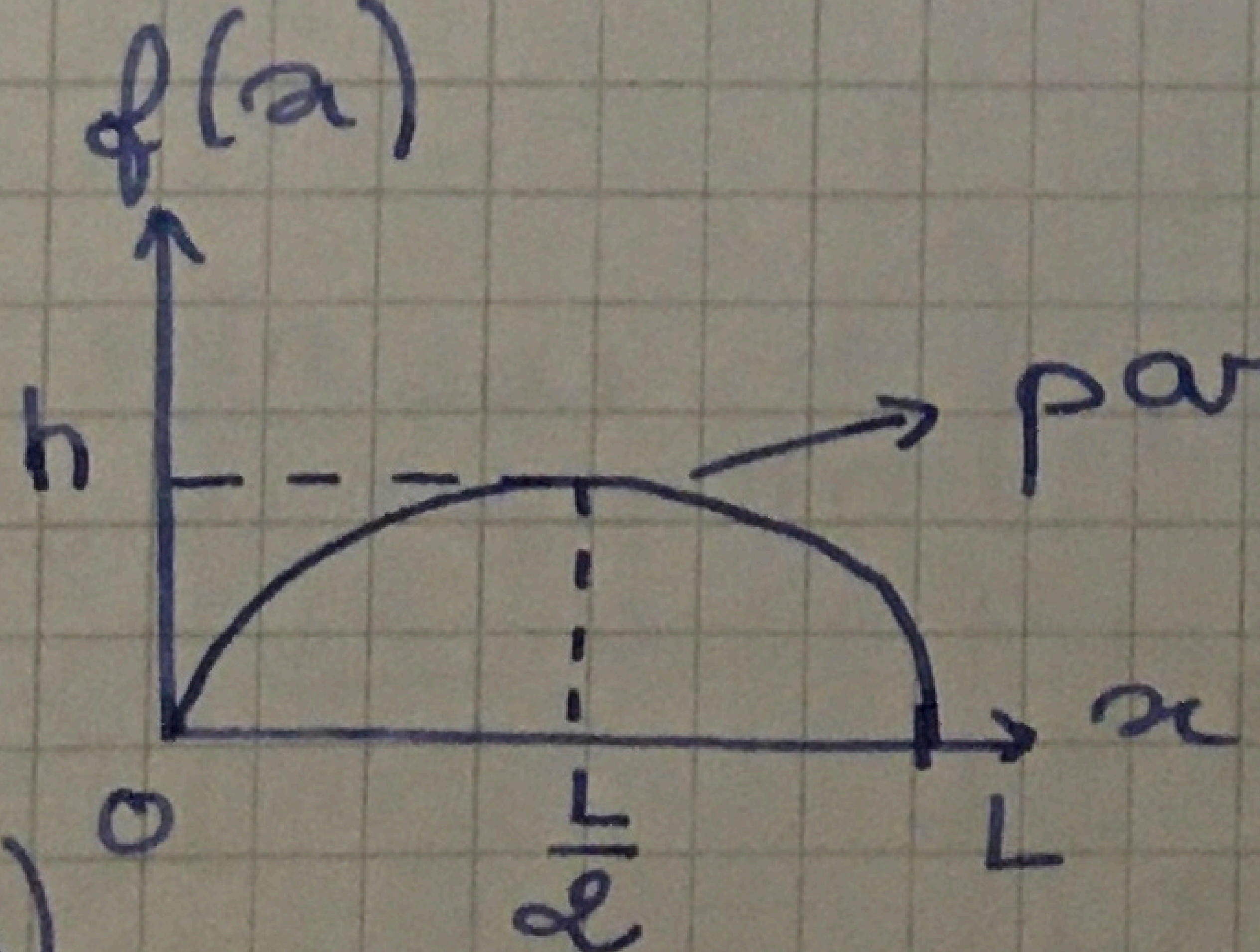
$$F_n = \frac{2}{L}\left[\frac{h}{a}\cancel{\left(-\frac{aL}{n\pi}\cos\frac{n\pi a}{L}\right)} + \frac{h}{a}\left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi a}{L} \right.$$

$$\left. + \frac{h}{L-a}\cancel{\left(\frac{L(L-a)}{n\pi}\cos\frac{n\pi a}{L}\right)} + \frac{h}{L-a}\left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi a}{L} \right]$$

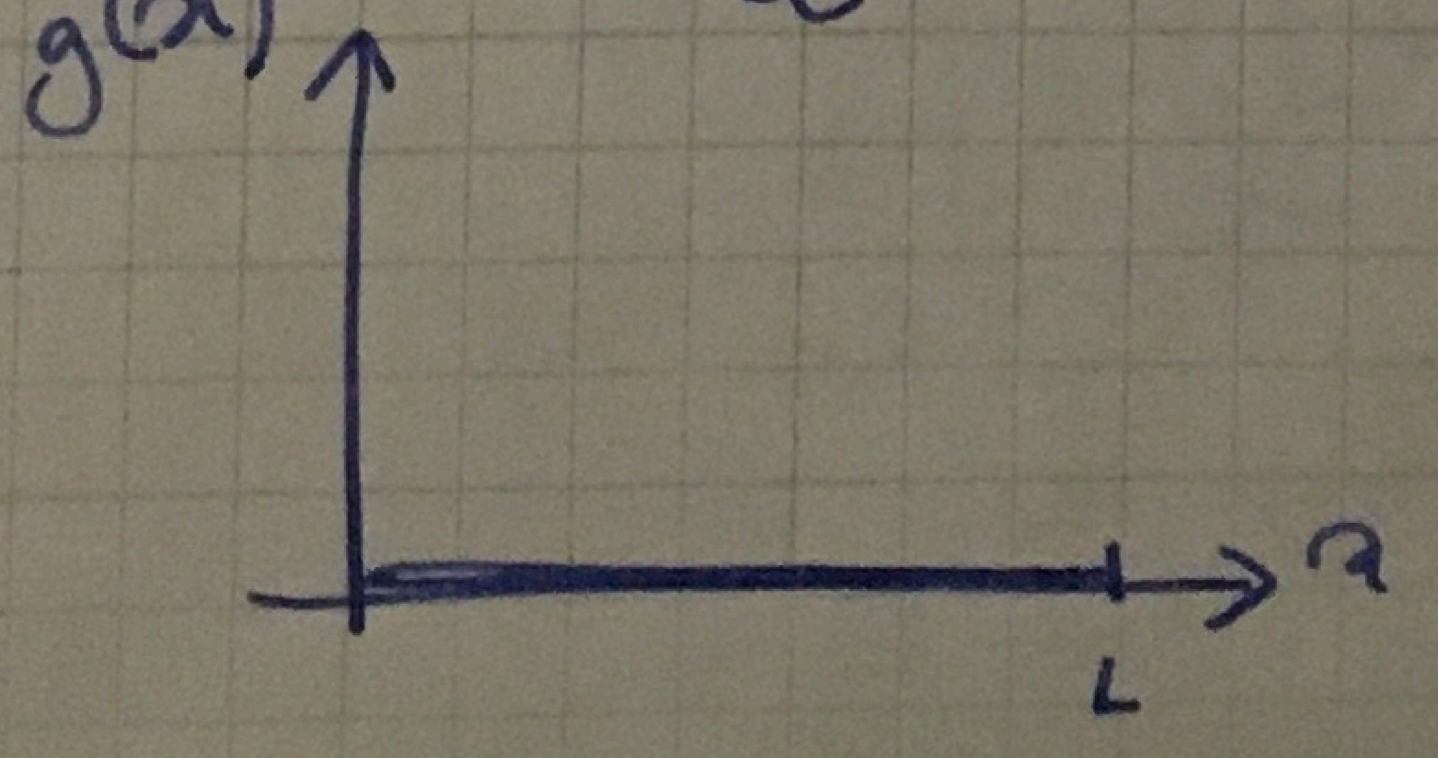
$$F_n = \frac{2}{L}\left(\frac{L}{n\pi}\right)^2 \underbrace{\left(\frac{h}{a} + \frac{h}{L-a}\right)}_{\frac{hL}{a(L-a)}} \sin\frac{n\pi a}{L}$$

$$\boxed{F_n = \frac{2h}{a(L-a)}\left(\frac{L}{n\pi}\right)^2 \sin\frac{n\pi a}{L}} \quad \propto \frac{1}{n^2}$$

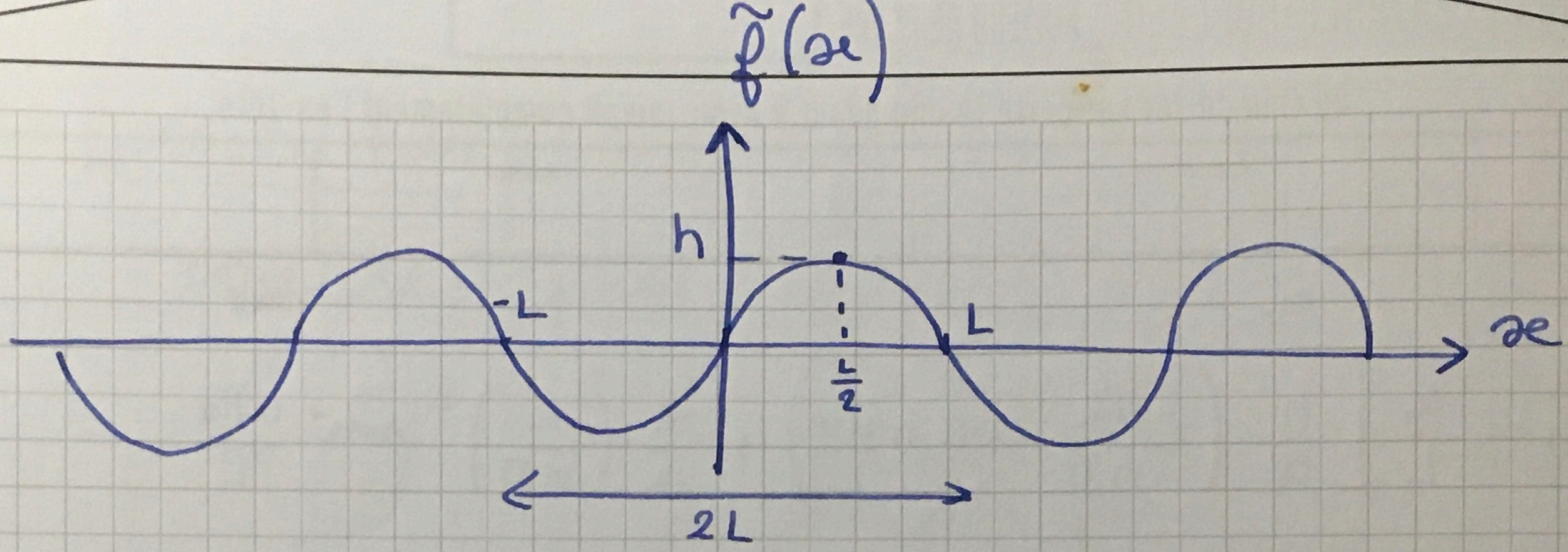
<u>Corde de Guitare</u> $\Rightarrow$ Corde pincée (parabole)



parabole $f(x) = \frac{4h}{L^2}(L-x)x$



$g(x) = 0 \Rightarrow B_n = 0$



Fonction 2L périodique impaire

$$\tilde{f}(x) = \sum_n F_n \sin\frac{n\pi x}{L}$$

$$F_n = \frac{4}{2L}\int_0^{\frac{2L}{2}} \tilde{f}(x)\sin\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{L}\int_0^L \frac{4h}{L^2}(L-x)x\,dx\,\sin\frac{n\pi x}{L}$$

$$= \frac{8h}{L^3}\left[\int_0^L Lx\sin\frac{n\pi x}{L}dx - \int_0^L x^2\sin\frac{n\pi x}{L}\,dx\right]$$

$$\int_0^L x\sin\frac{n\pi x}{L}\,dx = \left[-\frac{Lx}{n\pi}\cos\frac{n\pi x}{L}\right]_0^L + \int_0^L \frac{L}{n\pi}\cos\frac{n\pi x}{L}\,dx$$

$$= -\frac{L^2}{n\pi}\cos n\pi + \left(\frac{L}{n\pi}\right)^2 \underbrace{\left[\sin\frac{n\pi x}{L}\right]_0^L}_{0}$$

$$= -\frac{L^2}{n\pi}\cos n\pi$$

$$\int_0^L \underbrace{x^2}_{u} \underbrace{\sin\left(\frac{n\pi x}{L}\right)}_{v'} dx$$

$u \Downarrow u' = 2x$

$v' \Downarrow v = \frac{L}{n\pi}\left(-\cos\frac{n\pi x}{L}\right)$

$k_n = \frac{n\pi}{L}$

$\omega_n = \frac{n\pi c}{L}$

$f_n = \left(\frac{c}{2L}\right)n$

$$= \left[-x^2\frac{L}{n\pi}\cos\frac{n\pi x}{L}\right]_0^L + \int_0^L 2x\frac{L}{n\pi}\cos\frac{n\pi x}{L}\,dx$$

$$= -\frac{L^3}{n\pi}\cos n\pi + \frac{2L}{n\pi}\int_0^L \underbrace{x}_{u} \underbrace{\cos\frac{n\pi x}{L}}_{v'} dx$$

$u \Downarrow u' = 1$ $\quad v' \Downarrow v = \frac{L}{n\pi}\sin\frac{n\pi x}{L}$

$$\underbrace{\left[x\frac{L}{n\pi}\sin\frac{n\pi x}{L}\right]_0^L}_{0} - \int_0^L \frac{L}{n\pi}\sin\frac{n\pi x}{L}\,dx$$

$$= +\left(\frac{L}{n\pi}\right)^2\left(\cos n\pi - 1\right)$$

$$F_n = \frac{8h}{L^3}\left[\cancel{-\frac{L^3}{n\pi}\cos n\pi} + \cancel{\frac{L^3}{n\pi}\cos n\pi} - \frac{2L}{n\pi}\left(\frac{L}{n\pi}\right)^2\left(\cos n\pi - 1\right)\right]$$

$$F_n = \frac{8h}{L^3}\left(-\frac{2L^3}{n^3\pi^3}\left(\cos n\pi - 1\right)\right) \Rightarrow \begin{cases} F_{2p} = 0 \\ F_{2p+1} = \dfrac{32h}{\pi^3(2p+1)^3} \propto \dfrac{1}{n^3} \end{cases}$$

$p = 0 \Rightarrow f_0 = \frac{c}{2L} = 260\,Hz$ $\quad f_7 = 1800$

$p = 1 \Rightarrow f_3 = 3f_0 = 780\,Hz$ $\quad f_5 = 5f_0 = 1300\,Hz$

Conclusion

son le + pur : $\Downarrow$ $\boxed{\begin{array}{c}\text{Guitare}\\ \text{Clavecin}\\ \text{Piano}\end{array}}$

2) Les spectres st en contradiction avec la théorie

<u>Note</u>

$$\begin{cases} \text{Guitare} & f = 260\,Hz & Do_2 \\ \text{Piano} & f = 930\,Hz & La_4^{\#} \\ \text{clavecin} & f = 520\,Hz & Do_4. \end{cases}$$

<u>Piano</u>

$f_1 = 930\,Hz$ $\qquad B_1 = +\dfrac{2Ue}{\pi c}\sin\left(\dfrac{\pi a}{L}\right)$

$f_2 = 1850\,Hz$ $\qquad B_2 = +\dfrac{2Ue}{2\pi c}\sin\left(\dfrac{2\pi a}{L}\right)$

$$\frac{B_2}{B_1} = \frac{\sin 2\pi a/L}{2\sin(\pi a/L)} = \cos(2\pi a/L)$$

$$\left(\frac{0{,}014}{0{,}002}\right)^{-1} = \cos\left(\frac{2\pi a}{L}\right)$$

$$\frac{a}{L} = \frac{1}{2\pi}\arccos\left(\frac{0{,}002}{0{,}014}\right)$$

$$= 0{,}22$$